The zeta function of H^2 counting ideals

1 Presentation

H^2 has presentation

$$\langle x_1, x_2, y_1, y_2, z_1, z_2 \mid [x_1, y_1] = z_1, [x_2, y_2] = z_2 \rangle .$$

H^2 has nilpotency class 2.

2 The local zeta function

The local zeta function was first calculated by Grunewald, Segal & Smith. It is

$$\begin{aligned}\zeta^{\triangleleft}_{H^2,p}(s) = {} & \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-4)^2\zeta_p(5s-5) \\ & \times \zeta_p(5s-4)^{-1}.\end{aligned}$$

$\zeta^{\triangleleft}_{H^2}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\left.\zeta^{\triangleleft}_{H^2,p}(s)\right|_{p\to p^{-1}} = p^{15-10s}\zeta^{\triangleleft}_{H^2,p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{H^2}(s)$ is 4, with a simple pole at $s = 4$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta^{\triangleleft}_{H^2}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.